AP Review #7

(66) $f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5-x & \text{for } 3 < x \le 5 \end{cases}$

(a) Is f cont. at $x = 3$

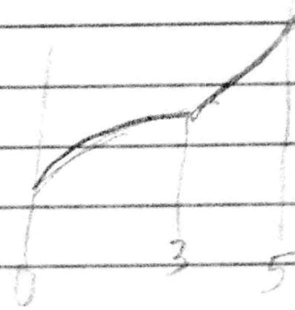
Cont. if $\lim\limits_{x \to 3} f(x) = f(3)$

$f(3) = \sqrt{3+1} = 2$

$\lim\limits_{x \to 3^-} f(x) = 2$ and $\lim\limits_{x \to 3^+} f(x) = 5-3 = 2$

$\therefore$ yes $f(x)$ is continuous at $x = 3$

(b) avg value of $f(x)$ on $0 \le x \le 5$



$\frac{1}{5}\left(\int_0^3 \sqrt{x+1}\,dx + \int_3^5 5-x\,dx\right)$

$= \frac{1}{5}\left(\frac{2}{3}(x+1)^{3/2}\Big|_0^3 + \left(5x - \frac{x^2}{2}\right)\Big|_3^5\right)$

$= \frac{1}{5}\left(\left(\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}\right) + \left(25 - \frac{25}{2}\right) - \left(15 - \frac{9}{2}\right)\right)$

$= \frac{1}{5}\left(\frac{16}{3} - \frac{2}{3} + \frac{25}{2} - \frac{21}{2}\right) = \frac{1}{5}\left(\frac{14}{3} + 2\right) = \frac{1}{5}\left(\frac{14}{3} + \frac{6}{3}\right)$

$= \frac{1}{5}\left(\frac{20}{3}\right) = \boxed{\frac{4}{3}}$

(c) $g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5 \end{cases}$

To be differentiable, the one-sided derivatives must be the same.

$\frac{k}{2}(x+1)^{-\frac{1}{2}} = m$ when $x = 3$, $g(x) = 2k$

at $x = 3$, $\frac{k}{2\sqrt{3+1}} = m$ and $m(3) + 2 = 2k$

$3m + 2 = 2k$

$k = \frac{3m+2}{2}$

$\frac{k}{4} = m$

$k = 4m$ $\frac{3m+2}{2} = 4m$

$3m + 2 = 8m$

$2 = 5m$ $\boxed{k = 4\left(\frac{2}{5}\right) = \frac{8}{5}}$

Calc (67) $f(x) = 3e^{2x}$ $g(x) = 6x^3$ (C)

$f'(x) = 6e^{2x}$ $g'(x) = 18x^2$

$6e^{2x} = 18x^2$ graph on calc $x = -.391$

(68) $\frac{dr}{dt} = -0.1$ cm/sec $C = 2\pi r$ $A = \pi r^2$

$r = \frac{C}{2\pi}$ $\frac{dA}{dt} = \cancel{2\pi} r \frac{dr}{dt}$

$= \cancel{2\pi}\left(\frac{C}{\cancel{2\pi}}\right)(-0.1)$

(B)

$\frac{dA}{dt} = -0.1C$

Calc (69) $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$ How many critical values does f have on (0, 10)

Critical values occur when $f'(x) = 0$ or DNE

$f'(x)$ DNE when $x = 0$ ~~but thats~~ not on (0, 10) open interval

Graph to see how many times $f'(x) = 0$ on (0, 10) → 3 (B)

(70) (A) f is NOT continuous at $x = a$

Calc (71) $f(x) = 4x^2 - x^3$ $l: y = 18 - 3x$ is tangent to graph of f

(a) show that l is tangent to graph of f(x) at point $x = 3$

$f'(x) = 8x - 3x^2$ slope of $l = -3$

$8x - 3x^2 = -3$

$3x^2 - 8x - 3 = 0$

$(3x + 1)(x - 3) = 0$

$x = -1/3$, $x = 3$ positive x-value intersection at point (3, 9)

(b) area of $S = \int_3^4 (18 - 3x) - (4x^2 - x^3)\,dx + \int_4^6 18 - 3x\,dx$

$4x^2 - x^3 = 0$

$x^2(4 - x) = 0$ $18 - 3x = 0$ $= \boxed{7.917}$

$x = 0, 4$ $-3x = -18$

$x = 6$

(c) Volume of R @ x-axis: $V = \pi \int_0^4 (4x^2 - x^3)^2\,dx = \boxed{490.208}$

(72) $f(x) = |x|$

I f is cont at $x=0$ TRUE

II f is diff. at $x=0$ FALSE

III f has abs min at $x=0$ TRUE

(D) I & III only

(73) f is cont. if $F'(x) = f(x)$ then $\int_1^3 f(2x)\,dx =$

$$\frac{1}{2}F(2x)\Big|_1^3 = \frac{1}{2}F(6) - \frac{1}{2}F(2)$$

(E)

(74) function has relative max if derivative Δs from + to −

f-yes, g-no, h-no

(A) f only

(75) If $\frac{dy}{dt} = ky$ and K is nonzero constant, then $y =$

$\int \frac{dy}{y} = \int k\,dt$

$\ln|y| = kt + C$

$|y| = \pm e^{kt+c}$

$y = \pm e^{kt} e^{c}$ $\quad y = \pm e^{c} e^{kt}$ $\quad$ let $A = \pm e^{c}$

So $y = Ae^{kt}$ $\quad$ (B) $2e^{kt}$ could be