AP REVIEW #5

(46) $\frac{dh}{dt} = -\frac{3}{10}$ cm/hr $\qquad V = \frac{1}{3}\pi r^2 h$

(a) $h = 2r$ (similar Δs $\Rightarrow$ proportion $\frac{r}{h} = \frac{5}{10}$)

When $h = 5$, $r = \frac{5}{2} = 2.5$ $\quad$ OR $r = \frac{h}{2}$

$$V = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2(5) = \frac{125\pi}{12}$$

(b) Find $\frac{dV}{dt}$ when $h = 5$cm

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{1}{3}\pi\frac{h^3}{4} = \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{4}(5)^2\left(-\frac{3}{10}\right) = -\frac{75\pi}{40}\text{ cm}^3/\text{hr}$$

OR $-\frac{15\pi}{8}$ cm³/hr

(c) $\frac{dV}{dt} = k\pi r^2 = \frac{\pi}{4}h^2\left(-\frac{3}{10}\right)$

$$k\pi r^2 = -\frac{3\pi}{40}(2r)^2$$

$$k\cancel{\pi}\cancel{r^2} = -\frac{3\cancel{\pi}}{\cancel{40}\,10}(\cancel{4}\cancel{r^2})$$

$$\boxed{k = -3/10}$$

(47) $\frac{dy}{dx} = y\sec^2 x \quad y = 5$ when $x = 0 \quad u = e$

$$\int\frac{dy}{y} = \int\sec^2 x\,dx$$

$$\ln|y| = \tan x + C$$

$$\ln 5 = C$$

$$\ln y = \tan x + \ln 5$$

$$y = e^{\tan x + \ln 5} = e^{\tan x}e^{\ln 5} = 5e^{\tan x} \quad \text{(C)}$$

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

(48) $f(x)=e^{-x^2}$ avg value of function on $[-1,1]$

$$=\frac{1}{1-(-1)}\int_{-1}^{1} e^{-x^2}dx=\frac{1}{2}\int_{-1}^{1} e^{-x^2}dx=.746824 \quad \text{(B)}$$

(49) $\frac{dy}{dx}=\frac{1}{x}$ avg rate of chg. of y w/respect to x on $[1,4]$

$y=\ln x \quad \frac{1}{3}\int_1^4 \frac{1}{x}dx=\frac{1}{3}(\ln 4-\ln 1)$

(C)

$\frac{1}{3}\ln 4$

$\frac{1}{3}\ln 2^2=\frac{2}{3}\ln 2$

(50) $\frac{dy}{dx}=\frac{1}{x+1}$ and $y(0)=0$ Euler's method w/ 2 steps $\Delta x=0.5$ to approx $y(1)$

	dy/dx	Δx	Δy	new pt
(0,0)	1	.5	.5	(.5,.5)
(.5,.5)	$\frac{2}{3}$	.5	$\frac{1}{3}$	$(1,\frac{1}{2}+\frac{1}{3})$

(D) $y(1)=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

(51) $\frac{dN}{dt}=0.05N-0.0005N^2$ N = # wolves t (years)

$P'=kP(M-P)$ $=.0005N(100-N)$ M = 100 (max) - carrying capacity

$\therefore \lim_{n\to\infty} N(t)=100$ (C)

(52) (a) $\int_0^{1.5}(3f'(x)+4)\,dx=3\int_0^{1.5}f'(x)dx+\int_0^{1.5}4\,dx$

$=3f(x)\Big|_0^{1.5}+4x\Big|_0^{1.5}=3(f(1.5)-f(0))+4(1.5)-0$

$=3(-1-(-7))+6=\boxed{24}$

(b) tang. to f when $x=1$ pt $(1,-4)$ $m=f'(1)=5$

$y+4=5(x-1)$ OR $y=5x-9$

$f(1.2)\approx 5(1.2)-9=6-9=-3$

$f(1.2)=-3$ would be less than actual b/c f is concave up on $-1.5<x<1.5$ since $f''(x)>0$

52 (cont'd)

c $f''(x) = \dfrac{f'(.5) - f'(0)}{.5 - 0} = \dfrac{3 - 0}{\frac{1}{2}} = 6 = r$ MVT states there must be a value between 0 and 0.5 where derivative value equals slope of secant

d $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \geq 0 \end{cases}$

g passes thru all points $(x, f(x))$ in table. Is it possible f and g are the same function?

for $x < 0$, $g'(x) = 4x - 1$ and $g''(x) = 4$

for $x \geq 0$, $g'(x) = 4x + 1$ and $g''(x) = 4$

<u>NO</u> b/c when $x = 0$, $f'(0) = 0$ which isn't true for g

$g'(0) = 1$.

53 $\lim\limits_{h \to 0} \dfrac{3\left(\frac{1}{2} + h\right)^5 - 3\left(\frac{1}{2}\right)^5}{h}$ $f(x) = 3x^5$ $f'(x) = 15x^4$ $f'\left(\frac{1}{2}\right) = \frac{15}{16}$

OR use L'Hopitals Rule

C

54 B $f'(c)$ might never equal zero on $a < c < b$

55 $\displaystyle\int_1^e \left(\frac{x^2 - 1}{x}\right) dx = \int_1^e x - \frac{1}{x}\, dx = \left(\frac{x^2}{2} - \ln x\right)\Big|_1^e$

$= \left(\frac{e^2}{2} - \ln e\right) - \left(\frac{1}{2} - \ln 1\right) = \frac{e^2}{2} - 1 - \frac{1}{2} = \frac{e^2}{2} - \frac{3}{2}$

E