Work the following on **notebook paper**.

PARAMETRIC MOTION

Multiple Choice

1. (2003 AB 25) (Noncalculator)
A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

$v(t) = 6t^2 - 42t + 72$
$= 6(t^2 - 7t + 12)$
$= 6(t-4)(t-3)$
$v(t) = 0$ at $t = 3, 4$

(A) $t = 1$ only (B) $t = 3$ only (C) $t = \frac{7}{2}$ only
(D) $t = 3$ and $t = \frac{7}{2}$ (E) $t = 3$ and $t = 4$

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2. (1998 AB 24) (Noncalculator)
The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

$a(t) = 3t^2 - 6t + 12$ $a'(t) = 6t$ $6(t$ test endp + critical p

$a(0) = 12$ $a(3) = 27 - 18 + 12 = 21$ $a(1) = 3 - 6 + 12 = 9$

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

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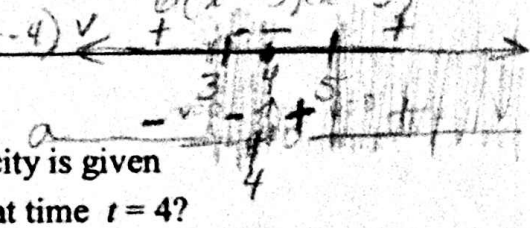
3. (Sample Multiple Choice AB 9) (Noncalculator)
The position of a particle moving along a line is given by $s(t) = 2t^3 - 24t^2 + 90t + 7$ for $t \geq 0$.
For what values of t is the speed of the particle increasing?

$v(t) = 6t^2 - 48t + 90$
$6(t^2 - 8t + 15)$
$6(t-3)(t-5)$
$a(t) = 12t - 48 = 12(t-4)$

(A) $3 < t < 4$ only (B) $t > 4$ only (C) $t > 5$ only
(D) $0 < t < 3$ and $t > 5$ (E) $3 < t < 4$ and $t > 5$



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4. (2003 AB 76) (Calculator)
A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.97

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5. (2003 AB 91) (Calculator)
A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$ If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

$v(2) = 2 + \int_1^2 \ln(1+2^x)\,dx$
$v(1)$

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 (E) 3.346

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6. (Sample Multiple Choice AB 19) (Calculator)
Two particles start at the origin and move along the x-axis. For $0 \leq t \leq 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

$v_{x_1} = \cos t$ $v_{x_2} = -2e^{-2t}$

(A) None (B) One (C) Two (D) Three (E) Four

7. (Sample Multiple Choice AB 15) (Calculator)

A particle travels along a straight line with a velocity is given of $v(t) = 3e^{(-t/3)}\sin(2t)$ meters per second. What is the total distance traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835 (B) 1.850 (C) 2.055 (D) 2.261 (E) 7.025

$\int_0^2 |v(x)| = \int_0^2 |3e^{-t/3}\sin(2$

Free Response

8. (2004 AB 3) (Calculator)

A particle moves along the y-axis so that its velocity at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1}x = \arctan x$)

y1 =

(a) Find the acceleration of the particle at time $t = 2$. $a(2) = v'(2) = \frac{d}{dt}(1-\tan^{-1}(e^t))\Big|_{t=2} = -.13$

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer. $v(2) = -.436$ Speed is increasing b/c $v(2)$ and $a(2)$ are both negative

(c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer. $t = .443$ b/c v

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

$s(2) = -1 + \int_0^2 (1-\tan^{-1}(e^t))\,dt = -1.361$ Moving away from origin b/c position is to left + $v(t) < 0$ meaning moving left

9. (2006 AB 4/BC4) (Noncalculator)

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

$s(t)$ 0

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(a) Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure. rate of change of $v(t)$

$a(t) = v'(t)$ = "slope of v" — average over 80 sec $= \frac{49-5}{80} = \frac{11}{20}$ ft/sec^2

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

the $\int_{10}^{70} v(t)\,dt$ represents distance, in feet, traveled by rocket from $t = 10$ sec to $t = 70$ sec.

$\int_{10}^{70} v(t)\,dt \approx 20(22 + 35 + 44) = 2020$ feet

(c) Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

Rocket A at $t = 80$ sec $(v_A) = 49$ ft/sec

$v_B(80) = 2 + \int_0^{80} (t+1)^{-1/2}\,dt = 2 + 3\left(2(t+1)^{1/2}\right)\Big|_0^{80}$

$= 2 + 3(2(9) - 2(1))$

$v_B(80) = 2 + 3(16) = 50$ ft/sec

$\therefore$ Rocket B is going faster b/c $50 > 49$.