AP Review

$\boxed{1A}$

① $g'(x) = f(x)$ $\boxed{g(x) = \int_1^x f(t)\,dt}$

ⓐ $g(4) = \int_1^4 f(t)\,dt = 2.5$ ⓑ $\underline{g'(1) = 4}$

$g(-2) = \int_1^{-2} f(t)\,dt = -6$

ⓒ Consider endpoints $g(-2) = -6$
$g(4) = 2.5$
and critical point $g(3) = 3$

∴ Abs. min value of $g = -6$ b|
-6 is smallest value of candidates

ⓓ POI only @ $x = 1$ b|c g'' Δs signs @ $x = 1$.

⑪ $\int x f(x)\,dx =$ $= \frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x)\,dx$

$\underline{\int u\,dv} = uv - \int v\,du$

$\boxed{B}$

$u = f(x)$ $dv = x\,dx$
$du = f'(x)\,dx$ $v = \frac{x^2}{2}$

③ $x^3 + 3x \cdot y + 2y^3 = 17$

$\underline{\underline{3x^2}} + 3x\frac{dy}{dx} \underline{+3y} + 6y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3x + 6y^2) = -3x^2 - 3y$ $\boxed{A}$

$\frac{dy}{dx} = \frac{-3x^2 - 3y}{3x + 6y^2} = \frac{-x^2 - y}{x + 2y^2} = -\frac{(x^2 + y}{x + 2y^2}$

④ $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}} + C = 2\sqrt{x^3+1} + C$

$u = x^3 + 1$
$du = 3x^2\,dx$

$\boxed{A}$

⑥ min value of $f(x) = x\ln x$ $f'(x) = x\frac{1}{x} + \ln x(1)$

$f' = 0$ or DNE $f'(x) = 1 + \ln x = 0$

f' as from − to + $\ln x = -1$

$e^{-1} = x$

$f\left(\frac{1}{e}\right) = \frac{1}{e}\ln e^{-1} = \frac{1}{e}(-1)$ $x = \frac{1}{e}$

$= \boxed{-\frac{1}{e}}$ $\boxed{C}$

⑦ $\frac{f(b)-f(a)}{b-a} = f'(c)$ $f(x) = \sin\left(\frac{x}{2}\right)$

$\frac{\pi}{2} < x < \frac{3\pi}{2}$

$\frac{0}{\pi} = \left(\cos\frac{x}{2}\right)\frac{1}{2}$ $\left(\frac{\pi}{2}, \frac{\sqrt{2}}{2}\right)$ $\left(\frac{3\pi}{2}, \frac{\sqrt{2}}{2}\right)$

$0 = \cos\frac{x}{2}$

$\frac{x}{2} = \frac{\pi}{2}$ $\boxed{x = \pi}$ $\boxed{D}$

⑧ $y = \frac{1}{x^2} - \frac{1}{x^3} = x^{-2} - x^{-3}$ $y' = -2x^{-3} + 3x^{-4}$

$y'' = 6x^{-4} - 12x^{-5}$

y'' DNE when $x = 0$

$y'' = 0$ @ $x = 2$

$y'' = \frac{6x}{x^4} - \frac{12}{x^5} =$

y'' ← + ∅ − 0 + →

0 2

6 $y'' = \frac{6x-12}{x^5}$ $\frac{6}{x^4} = \frac{12}{x^5}$

$6x^5 = 12x^4$

$6x^5 - 12x^4 = 0$

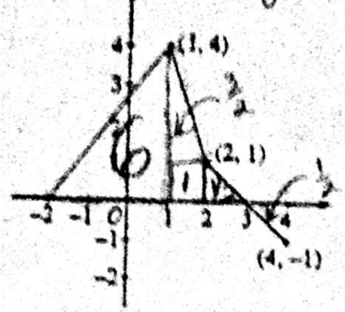
$6x^4(x-2) = 0$

AP REVIEW #1A

Work the following on notebook paper, showing all work. **No calculator**.

1. The graph of the function f, consisting of three line segments, is shown. Let $g(x)=\int_1^x f(t)\,dt$.

$f(x)=g'(x)$



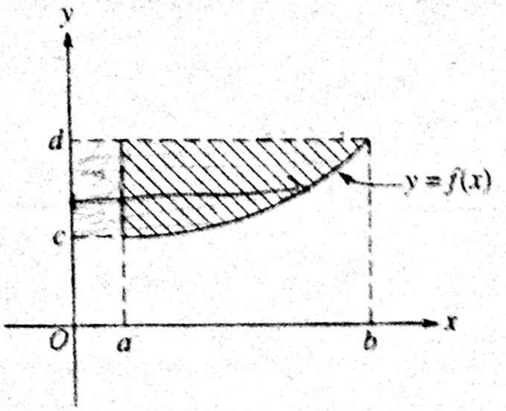
(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x=1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x=1$ and $x=2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

2.



Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)\,dy$ [✓ (B) $\int_a^b (d-f(x))\,dx$] ~~(C) $f'(b)-f'(a)$~~

~~(D) $(b-a)[f(b)-f(a)]$~~ ~~(E) $(d-c)[f(b)-f(a)]$~~

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3. If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$ $3x^2+3x\frac{dy}{dx}+3y+6y^2\frac{dy}{dx}=0$

[✓ (A) $\frac{x^2+y}{x+2y^2}$] (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$ (D) $-\frac{x^2+y}{2y^2}$ (E) $\frac{-x^2}{1+2y^2}$

$\frac{dy}{dx}(3x+6y^2)=-3x$

$\frac{dy}{dx}=\frac{-x^2-y}{x+2}$

$u=x^3+1$ 4. $\int\frac{3x^2}{\sqrt{x^3+1}}dx=$ $\int\frac{du}{\sqrt{u}}=\int u^{-1/2}du=2u^{1/2}+C=2\sqrt{x^3+1}+C$

$du=3x^2dx$

$dx=\frac{du}{3x^2}$

[✓ (A) $2\sqrt{x^3+1}+C$] (B) $\frac{3}{2}\sqrt{x^3+1}+C$ (C) $\sqrt{x^3+1}+C$ (D) $\ln\sqrt{x^3+1}+C$ (E) $\ln(x^3+1)+C$

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5. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ [✓ (D) $\frac{7}{3}$] (E) $\frac{5}{2}$

$f'=0$ or DNE

f' ∆s from + to −

$f'(x)=(x-2)2(x-3)+(x-3)^2=0$

$(x-3)(2(x-2)+(x-3))=0$

$2x-4+x-3=0$

$3x=7$ $x=\frac{7}{3}$

$\frac{7}{3}$

6. Let f be a function with $f(1)=4$ such that for all points (x, y) on the graph of f the slope is given by $\frac{3x^2+1}{2y}$. $= \frac{dy}{dx}$ $(1,4)$

(a) Find the slope of the graph of f at the point where $x=1$. $\frac{4}{8}$ $\boxed{\frac{1}{2}}$

(b) Write an equation for the line tangent to the graph of f at $x=1$, and use it to approximate $f(1.2)$.

$y-4=\frac{1}{2}(x-1)$ $y-4=\frac{1}{2}(.2)$ $y=.1+4=\boxed{4.1}$

(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx}=\frac{3x^2+1}{2y}$ with the initial condition $f(1)=4$.

$\int 2y\,dy=\int 3x^2+1\,dx$ $y^2=x^3+x+C$

$16=2+C$ $C=14$

(d) Use your solution from part (c) to find $f(1.2)$.

$f(x)=\sqrt{x^3+x+14}$ $f(1.2)=\sqrt{(1.2)^3+1.2+14}$ $\boxed{4.114}$

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7. If $f(x)=\sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2}<x<\frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

$\frac{f(b)-f(a)}{b-a}=f'($

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ [✓ (D) π] (E) $\frac{3\pi}{2}$

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8. If $f(x)=(x-1)^2\sin x$, then $f'(0)=$

$f'(x)=(x-1)^2\cos x+\sin x(2($

(A) -2 (B) -1 (C) 0 [✓ (D) 1] (E) 2

$f'(0)=1\cdot1+0$

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9. The acceleration of a particle moving along the x-axis at time t is given by $a(t)=6t-2$. If the velocity is 25 when $t=3$ and the position is 10 when $t=1$, then the position $x(t)=$

(A) $9t^2+1$ (B) $3t^2-2t+4$ [✓ (C) t^3-t^2+4t+6] (D) $t^3-t^2+9t-20$ (E) $36t^3-4t^2-77t+55$

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10. $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ [✓ (D) $\cos(2\pi x)$] (E) $2\pi\cos(2\pi x)$

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11. $\int x\cdot f(x)\,dx=$

$\int u\,dv=uv-\int v\,du$ $u=f(x)$ $dv=x\,dx$ $du=f'(x)dx$ $v=\frac{x^2}{2}+C$

(A) $x f(x)-\int x f'(x)dx$ [✓ (B) $\frac{x^2}{2}f(x)-\int\frac{x^2}{2}f'(x)dx$] (C) $x f(x)-\frac{x^2}{2}f(x)+C$

(D) $x f(x)-\int f'(x)dx$ (E) $\frac{x^2}{2}\int f(x)dx$

$\frac{x^2}{2}f(x)-\int f'(x)\frac{x^2}{2}dx$

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12. What is the minimum value of $f(x)=x\ln x$?

(A) $-e$ (B) -1 [✓ (C) $-\frac{1}{e}$] (D) 0 (E) $f(x)$ has no minimum value.

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13. At what value of x does the graph of $y=\frac{1}{x^2}-\frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 [✓ (C) 2] (D) 3 (E) At no value of x