AP Review 4

OR $\int_0^{1.266\ldots} \sqrt[3]{2-y} - \tan^{-1} y \, dy$

(34) (a) Area of R $= \int_0^B \tan x \, dx + \int_B^{\sqrt[3]{2}} 2 - x^3 \, dx = \boxed{.729}$

$\tan x = 2 - x^3$
at $x = .902155 \rightarrow$ STO B
$2 - x^3 = 0$ at $x = \sqrt[3]{2}$

(b) Area of S $= \int_0^B (2 - x^3 - \tan x) \, dx = \boxed{1.161}$

(c) Volume when S revolved about x-axis:

$$V = \pi \int_0^B (2-x^3)^2 - (\tan x)^2 \, dx = 2.652\pi = 8.332$$

(35) $f(1) = 2 \quad f(3) = 7$ on interval $1 \le x \le 3$

I. average rate of change of $f = \frac{f(3) - f(1)}{3-1} = \boxed{\frac{5}{2}}$ ✓ TRUE

II. average value of $f = \frac{1}{2} \int_1^3 f(x) \, dx$ - Don't Know

III average value of $f' = \frac{1}{2} \int_1^3 f'(x) \, dx = \frac{1}{2}(f(3) - f(1)) = \boxed{\frac{5}{2}}$ ✓ TRUE

$\boxed{D}$ I & III only

(36) $\int \frac{dx}{(x-1)(x+3)} = \frac{1}{4}\left(\int \frac{1}{x-1} dx - \int \frac{1}{x+3} dx\right)$

$= \frac{1}{4}(\ln|x-1| - \ln|x+3|) + C$

$= \frac{1}{4} \ln\left|\frac{x-1}{x+3}\right| + C \quad \boxed{A}$

$\frac{1}{(x-1)(x+3)} = \frac{A(x+3)}{(x-1)(x+3)} + \frac{B(x-1)}{(x+3)(x-1)}$

$1 = A(x+3) + B(x-1)$
if $x = -3$: $1 = -4B \quad B = -\frac{1}{4}$
if $x = 1$: $1 = 4A \quad A = \frac{1}{4}$

(37)

$y = 2 - x^2$
$y - 2 = -x^2$
$2 - y = x^2$
$x = \sqrt{2-y}$

$V = \int_0^2 (\sqrt{2-y})^2 \, dy = \boxed{B} \quad \int_0^2 (2-y) \, dy$

(38) $\lim_{h\to 0} \frac{\tan 3(x+h) - \tan(3x)}{h} = \frac{d}{dx}\tan 3x = \sec^2 3x(3)$ $\boxed{B}$

(39) $F(x) = \cos(2x) + e^{-x}$

Find c such that $F'(c) = \frac{F(3) - F(0)}{3-0}$

let $Y1 = F(x) = \cos(2x) + e^{-x}$

graph Y2 & Y3 & find intersection $\begin{cases} Y2 = \frac{d}{dx}(Y1)_{x=x} \longrightarrow F'(x) \\ Y3 = A \end{cases}$

$\frac{F(3) - F(0)}{3} = \frac{\cos(6) + e^{-3} - 2}{3} = -.7300... \rightarrow$ STO A

$Y2\,(F'(x))$ and $Y3\left(\frac{F(3)-F(0)}{3-0}\right)$ are equal at $x = 1.542$

Answer $\boxed{A}$

(40) $\frac{dy}{dx} = y^2(6-2x)$

ⓐ $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}(6-2x) - 2y^2 = 2y(y^2(6-2x))(6-2x) - 2y^2$

$= 2y^3(6-2x)(6-2x) - 2y^2$

$\left.\frac{d^2y}{dx^2}\right|_{(3,\frac{1}{4})} = 2\left(\frac{1}{64}\right)(0)(0) - 2\left(\frac{1}{16}\right) = \boxed{-\frac{1}{8}}$

ⓑ $\int \frac{dy}{y^2} = \int (6-2x)\,dx$

$-\frac{1}{y} = 6x - x^2 + C$

$(3, \frac{1}{4})$ $-4 = 18 - 9 + C$

$C = -13$

$-\frac{1}{y} = 6x - x^2 - 13$

$\boxed{y = \frac{1}{x^2 - 6x + 13}}$

(41) $f(3)=2$ $f'(3)=5$

tangent to graph of f at $x=3$: $y-2=5(x-3)$

$y=5x-13$

Use tangent to approximate zero of f: $0=5x-13$

$\frac{13}{5}=x=2.6$ $\boxed{C}$

(42) Trap Approx $= \frac{.5}{2}\left(3+2(3)+2(5)+2(8)+13\right)$

$= \frac{1}{4}\left(3+16+16+13\right)=\frac{1}{4}(48)=12$ $\boxed{B}$

(43) $f'(x)=\frac{x^2}{1+x^5}$ and $f(1)=3$, then $f(4)=$

$f(4)=f(1)+\int_1^4 f'(x)\,dx = 3+\int_1^4 \frac{x^2}{1+x^5}dx = 3.316$

$\boxed{D}$

(44) $F'(x)=f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx=$

$\int_1^3 f(2x)dx = \frac{1}{2}(F(2x))\Big|_1^3 = \frac{1}{2}\left(F(6)-F(2)\right)=\frac{1}{2}F(6)-\frac{1}{2}F(2)$ $\boxed{E}$

(45) RRAM $= 10(400)+15(350)+12(280)+9(200)+11(180)$

$\boxed{C}$ $=16,930$ gal