of the following, sketch a graph, shade the region, and find the area.
e your calculator.

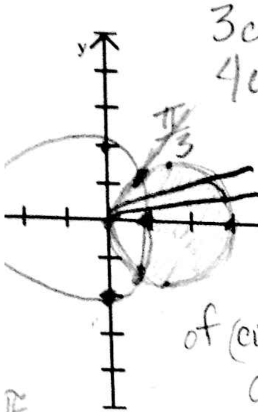
$r = 3\cos\theta$ and outside $r = 2 - \cos\theta$

$3\cos\theta = 2 - \cos\theta$

$4\cos\theta = 2$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$



Integrate from 0 to $\frac{\pi}{3}$ of $(\text{circle})^2 - (\text{cardioid})^2$ and double it.

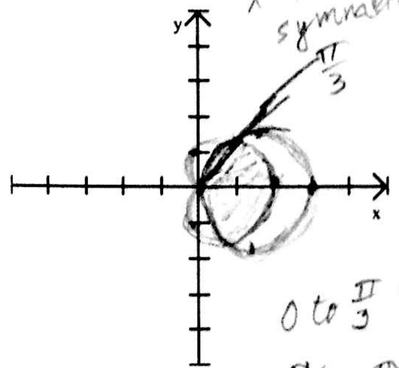
$\int_0^{\frac{\pi}{3}} 9\cos^2\theta - (2-\cos\theta)^2\, d\theta$

$9 - 4 + 4\cos\theta + \cos^2\theta\, d\theta$

$\frac{1+\cos 2\theta}{2}) - 4 + 4\cos\theta$

4. common interior of $r = 3\cos\theta$ and $r = 1 + \cos\theta$

X-axis symmetry



$3\cos\theta = 1 + \cos\theta$

$2\cos\theta = 1$

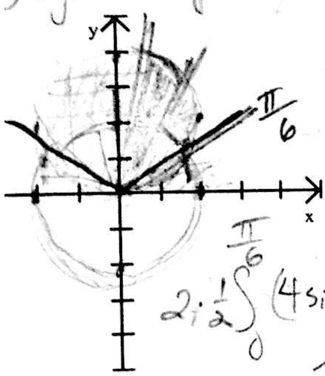
$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

0 to $\frac{\pi}{3}$ inside cardiod
$\frac{\pi}{3}$ to $\frac{\pi}{2}$ inside circle
then double

on interior of $r = 4\sin\theta$ and $r = 2$

s symmetry



$4\sin\theta = 2$

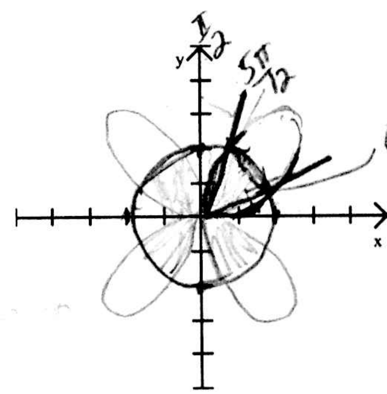
$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$2 \cdot \frac{1}{2}\int_0^{\frac{\pi}{6}} (4\sin\theta)^2\, d\theta$

$+ 2\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2^2\, d\theta$

5. common interior of $r = 4\sin(2\theta)$ and $r = 2$



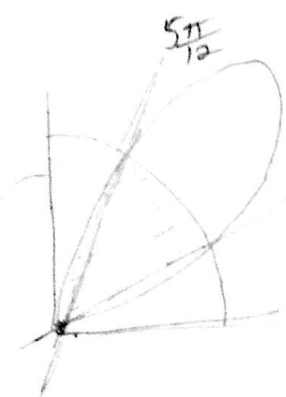
QI area times 4

$4\sin(2\theta) = 2$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

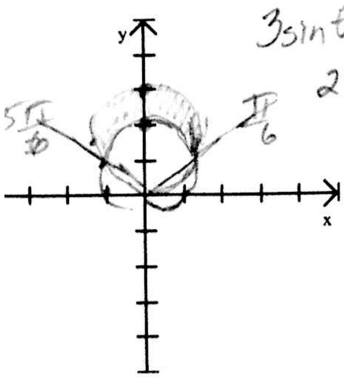
$\theta = \frac{\pi}{12}, \frac{5\pi}{12}$



0 to $\frac{\pi}{12}$ inside $(4\sin(2\theta))^2$ doubled

$\frac{\pi}{12}$ to $\frac{5\pi}{12}$ inside $r^2 (2)^2$

le $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$



$3\sin\theta = 1 + \sin\theta$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Polar Worksheet 3

① $A = 2\cdot\frac{1}{2}\int_0^{\pi/3} 9\cos^2\theta - (2-\cos\theta)^2\,d\theta$

$= \int_0^{\pi/3} 9\cos^2\theta - 4 + 4\cos\theta - \cos^2\theta\,d\theta$

$= \int_0^{\pi/3} \left(8\left(\frac{1+\cos 2\theta}{2}\right) - 4 + 4\cos\theta\right)d\theta$

$= \int_0^{\pi/3} 4\cos 2\theta + 4\cos\theta\,d\theta$

$= 4\int_0^{\frac{\pi}{3}} \cos 2\theta + \cos\theta\,d\theta$

$= 4\left(\frac{1}{2}\sin 2\theta + \sin\theta\right)\Big|_0^{\pi/3}$

$= 4\left[\left(\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}\right) - 0\right]$

$= 4\left(\frac{\sqrt{3}}{4} + \frac{2\sqrt{3}}{4}\right) = \boxed{3\sqrt{3}}$

② $A = 2\cdot\frac{1}{2}\int_0^{\pi/6} (4\sin\theta)^2\,d\theta + 2\left(\frac{1}{2}\right)\int_{\pi/6}^{\frac{\pi}{2}} 2^2\,d\theta$

$= \int_0^{\pi/6} 16\left(\frac{1-\cos 2\theta}{2}\right)d\theta + 4\theta\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \int_0^{\pi/6} 8 - 8\cos 2\theta\,d\theta + \left(2\pi - \frac{2\pi}{3}\right)$

$= (8\theta - 4\sin 2\theta)\Big|_0^{\frac{\pi}{6}} + \frac{4\pi}{3}$

$= \frac{4\pi}{3} - 4\left(\frac{\sqrt{3}}{2}\right) + \frac{4\pi}{3}$

$= \frac{8\pi}{3} - \frac{4\sqrt{3}}{2} = \boxed{\frac{8\pi}{3} - 2\sqrt{3}}$

(3) $A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (3\sin\theta)^2 - (1+\sin\theta)^2 \, d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(9\sin^2\theta - 1 - 2\sin\theta - \sin^2\theta\right) d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(8\left(\frac{1-\cos 2\theta}{2}\right) - 1 - 2\sin\theta\right) d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} 3 - 4\cos 2\theta - 2\sin\theta \, d\theta$

$= \frac{1}{2}\left(3\theta - 2\sin 2\theta + 2\cos\theta\right)\Big|_{\pi/6}^{5\pi/6}$

$= \frac{1}{2}\left[\left(\frac{15\pi}{6} - \cancel{2\left(\frac{\sqrt{3}}{2}\right)} + \cancel{2\left(-\frac{\sqrt{3}}{2}\right)}\right) - \left(\frac{3\pi}{6} - \cancel{2\left(\frac{\sqrt{3}}{2}\right)} + \cancel{2\frac{\sqrt{3}}{2}}\right)\right]$

$= \frac{1}{2}\left(\frac{12\pi}{6}\right) = \boxed{\pi}$

(4) $A = 2\cdot\frac{1}{2}\int_{0}^{\pi/3} (1+\cos\theta)^2 \, d\theta + 2\cdot\frac{1}{2}\int_{\pi/3}^{\pi/2} (3\cos\theta)^2 \, d\theta$

$= \int_{0}^{\pi/3} (1 + 2\cos\theta + \cos^2\theta) \, d\theta + \int_{\pi/3}^{\pi/2} 9\cos^2\theta \, d\theta$

$= \int_{0}^{\pi/3} \left(1 + 2\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta + \int_{\pi/3}^{\pi/2} 9\left(\frac{1+\cos 2\theta}{2}\right) d\theta$

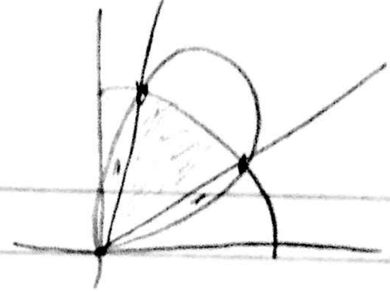
$= \int_{0}^{\pi/3} \left(\frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta + \frac{9}{2}\int_{\pi/3}^{\pi/2} (1 + \cos 2\theta) \, d\theta$

$= \left(\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right)\Big|_{0}^{\pi/3} + \frac{9}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right)\Big|_{\pi/3}^{\pi/2}$

$= \frac{\pi}{2} + 2\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right) + \frac{9}{2}\left[\left(\frac{\pi}{2} + 0\right) - \left(\frac{\pi}{3} + \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$

$= \frac{\pi}{2} + \cancel{\frac{8\sqrt{3}}{8}} + \cancel{\frac{\sqrt{3}}{8}} + \frac{9\pi}{4} - \frac{9\pi}{6} - \cancel{\frac{9\sqrt{3}}{8}}$

$= \frac{6\pi}{12} + \frac{27\pi}{12} - \frac{18\pi}{12} = \frac{15\pi}{12} = \boxed{\frac{5\pi}{4}}$



(5) $\frac{1}{4}A = 2\cdot\frac{1}{2}\int_0^{\pi/12} 16\sin^2(2\theta)\,d\theta + \frac{1}{2}\int_{\pi/12}^{5\pi/12} 4\,d\theta$

$\frac{1}{4}A = 8\int_0^{\pi/12} 1-\cos 4\theta \; d\theta + 2\theta\Big|_{\pi/12}^{5\pi/12}$

$\frac{1}{4}A = 8\left(\theta - \frac{1}{4}\sin 4\theta\right)\Big|_0^{\pi/12} + \left(\frac{5\pi}{6} - \frac{\pi}{6}\right)$

$\frac{1}{4}A = 8\left(\frac{\pi}{12} - \frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)\right) + \frac{4\pi}{6}$

$\frac{1}{4}A = \frac{16\pi}{12} - \sqrt{3}$

$A = \boxed{\frac{16\pi}{3} - 4\sqrt{3}}$

PAGE 2 OF WORKSHEET 3

(1) Find slope of curve of $r = 2-3\sin\theta$ at $(2, \pi)$ $\quad \frac{dr}{d\theta} = -3\cos\theta$

$m = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{(2-3\sin\theta)\cos\theta - 3\cos\theta\sin\theta}{-(2-3\sin\theta)\sin\theta - 3\cos\theta\cos\theta}$ $\quad x = (2-3\sin\theta)\cos\theta$

$y = (2-3\sin\theta)\sin\theta$

$\frac{dy}{dx}\Big|_{\theta=\pi} = \frac{2(-1) - 0}{-2(0) - 3(-1)(-1)} = \frac{-2}{-3} = \boxed{\frac{2}{3}}$

(2) Find tangent line to curve $r = 3\sin(2\theta)$ at pt where $\theta = \frac{\pi}{3}$

$m = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{(3\sin 2\theta)\cos\theta + 6\cos 2\theta\sin\theta}{-(3\sin 2\theta)\sin\theta + 6\cos 2\theta\cos\theta}$ $\quad x = (3\sin 2\theta)\cos\theta$

$y = (3\sin 2\theta)\sin\theta$

$m\Big|_{\theta=\frac{\pi}{3}} = \frac{3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + 6\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{-3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + 6\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)} = \frac{\frac{3\sqrt{3}}{4} - \frac{6\sqrt{3}}{4}}{-\frac{9}{4} - \frac{6}{4}}$ $\quad \frac{dr}{d\theta} = 6\cos 2\theta$

$x = 3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{3\sqrt{3}}{4}$

$m = -\frac{3\sqrt{3}}{4}\cdot -\frac{4}{15} = \frac{\sqrt{3}}{5}$ $\quad \boxed{y - \frac{9}{4} = \frac{\sqrt{3}}{5}\left(x - \frac{3\sqrt{3}}{4}\right)}$ $\quad y = 3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{9}{4}$

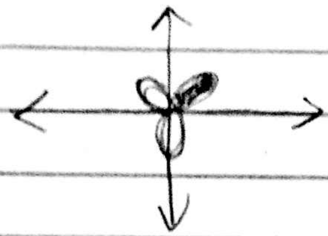
(3) $A = \frac{1}{2}\int_0^{\pi} \theta^2\,d\theta$

(4) $A = \frac{1}{2}\int_{\pi/2}^{\pi} (1+\sin\theta)^2\,d\theta$

(5) $A = \frac{1}{2}\int^{\pi/4} (2\sin 4\theta)^2\,d\theta$

⑥ $A = \frac{1}{2}\int_0^{\frac{\pi}{3}} \sin^2(3\theta)\, d\theta$

$r = \sin(3\theta)$



⑦ line $x = 1 \rightarrow r\cos\theta = 1$

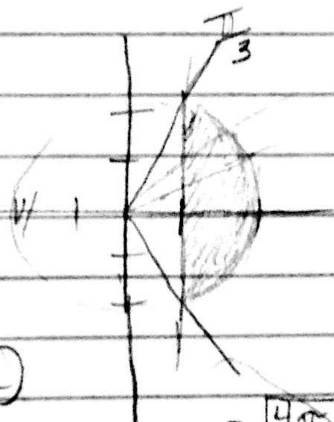
ⓐ $r = \frac{1}{\cos\theta}$

$\boxed{r = \sec\theta}$

circle w/ $r=2$ centered @ origin:

$\boxed{r = 2}$

ⓑ



$\sec\theta = 2$

$\cos\theta = \frac{1}{2}$

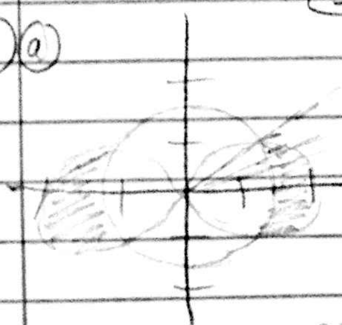
$\theta = \frac{\pi}{3}$

$A = 2 \cdot \frac{1}{2}\int_0^{\frac{\pi}{3}} 2^2 - \sec^2\theta\, d\theta$

ⓒ $= (4\theta - \tan\theta)\Big|_0^{\pi/3}$

$= \boxed{\frac{4\pi}{3} - \sqrt{3}}$

⑧ ⓐ



$r^2 = 4\cos 2\theta$

$r = \sqrt{2}$

$\sqrt{4\cos 2\theta} = \sqrt{2}$

$\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3},\ \theta = \frac{\pi}{6}$

$A = 4 \cdot \frac{1}{2}\int_0^{\frac{\pi}{6}} 4\cos 2\theta - 2\, d\theta$

$= 2\left(2\sin 2\theta - 2\theta\right)\Big|_0^{\frac{\pi}{6}}$

$= 2\left(2\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{3}\right)$

$= \boxed{2\sqrt{3} - \frac{2\pi}{3}}$

⑨ $r = \theta$ and $r = 2\theta$

Find area between spirals from $0 \le \theta \le 2\pi$

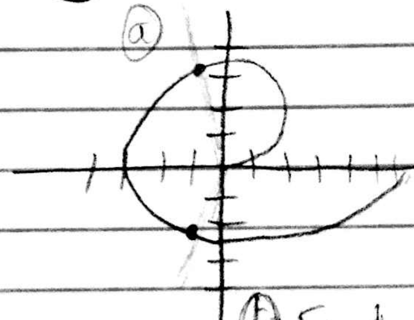
$A = \frac{1}{2}\int_0^{2\pi} (2\theta)^2 - \theta^2\, d\theta$

$= \frac{1}{2}\int_0^{2\pi} 3\theta^2\, d\theta = \frac{3}{2}\int_0^{2\pi} \theta^2\, d\theta$

$= \frac{3}{2} \cdot \frac{1}{3}\theta^3\Big|_0^{2\pi} = \frac{1}{2}(2\pi)^3 = \boxed{4\pi^3}$

⑩ $r = \theta + 2\sin\theta \quad 0 \le \theta \le 2\pi$

ⓐ



ⓑ Find θ for point(s) where $x = -1$

$r\cos\theta = -1$

$r = -\frac{1}{\cos\theta}$

when is $\theta + 2\sin\theta = -\frac{1}{\cos\theta}$?