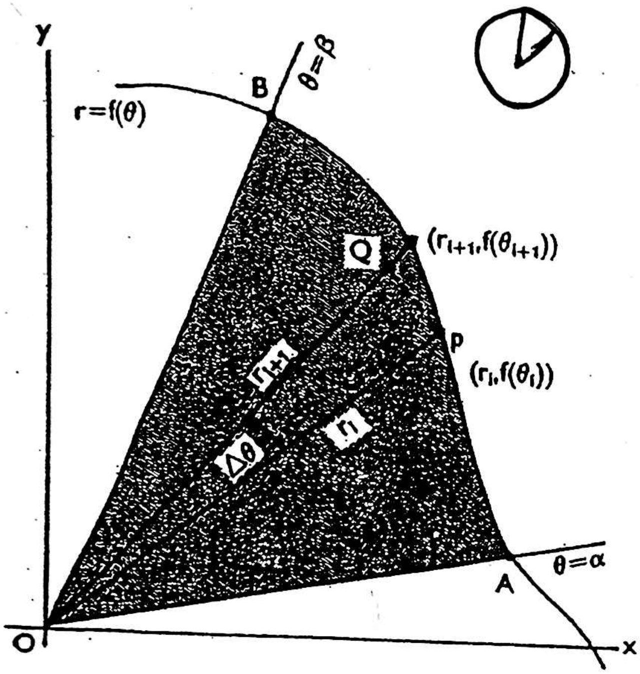
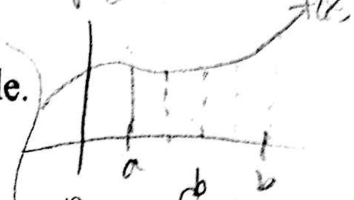
Day 2

Before we learned how to find the exact area under a function, we did sums of areas of rectangles or trapezoids.

The area of a polar region is based on the area of a sector of a circle.





$$\boxed{\text{Area of Sector} = \frac{\theta}{2\cancel{\pi}}\cancel{\pi} r^2}$$

$\text{Area} = \int_a^b f(x)\,dx$

so Area of Sector $= \frac{1}{2}r^2\theta$

$$\frac{\text{Sector Area}}{\text{Circle Area}} = \frac{\theta}{2\pi}$$

If we take a function $r = f(\theta)$ on $[\alpha, \beta]$ and partition it into equal subintervals, then

$\text{Sector Area} = \frac{\theta}{2\pi}\pi r^2$

radius of kth sector $= f(\theta_k)$

central angle of kth sector $= \dfrac{\beta-\alpha}{n} = \Delta\theta$

$$\text{Area} \approx \sum_{k=1}^{n} \frac{1}{2}[f(\theta_k)]^2 \bullet \Delta\theta$$

to find exact area, let $n \to \infty$ or let $\Delta\theta \to 0$

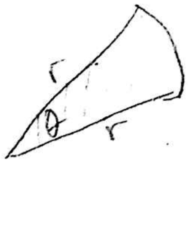
Taking the limit as $n \to \infty$,

$$\text{Area} = \lim_{n\to\infty} \frac{1}{2}\sum_{K=1}^{n}[f(\theta)]^2\Delta\theta = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2\,d\theta$$

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, then the area of the region bounded by the graph of $r = f(\theta)$, then the area of the region bounded by the graph of $r = f(\theta)$ and the radial lines $\theta = \alpha$ and $\theta = \beta$ is given by

$$\boxed{\text{Area} = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta.}$$ Formula Sheet

$$\frac{\theta}{2\pi} = \frac{\text{Sector Area}}{\pi r^2}$$



Sector Area $= \frac{\theta}{2\cancel{\pi}} \cdot \cancel{\pi} r^2 = \frac{1}{2}r^2\theta$

We need to add up all the little "slivers" (partitions of sector) by splitting up θ

Area of 1 sliver $= \frac{1}{2}r^2\Delta\theta$