From 2008 BC Multiple Choice

1. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \frac{45}{2} \right\rangle$ (B) $\langle 6, 5 \rangle$ (C) $\langle 2, 0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

$a(t) = \langle 2t, 5 \rangle$ $a(3) = \langle 6, 5 \rangle$

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5. Which of the following gives the length of the path described by the parametric equations $x = \sin(t^3)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

(A) $\int_0^{\pi} \sqrt{\sin^2(t^3) + e^{10t}}\, dt$ (B) $\int_0^{\pi} \sqrt{\cos^2(t^3) + e^{10t}}\, dt$ (C) $\int_0^{\pi} \sqrt{9t^4 \cos^2(t^3) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2 \cos(t^3) + 5e^{5t}}\, dt$ (E) $\int_0^{\pi} \sqrt{\cos^2(t^3) + e^{10t}}\, dt$

$x' = 3t^2 \cos(t^3)$ $y' = 5e^{5t}$ $L = \int_0^{\pi} \sqrt{(x')^2 + (y')^2}\, dt$

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28. In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt} > 0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2, 2)$?

(A) $\frac{2}{3}$ (B) $\frac{2\sqrt{10}}{3}$ (C) 3 (D) $\boxed{6}$ (E) $6\sqrt{10}$

$\frac{dy}{dt} = 2x\frac{dx}{dt} - \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{dx}{dt}(2x-1)$

$x=2$ so $dy/dt = dx/dt\,(3)$

speed $= 2\sqrt{10} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\left(\frac{dy}{dt}\right)^2 = 9\left(\frac{dx}{dt}\right)^2$ $2\sqrt{10} = \sqrt{\left(\frac{dx}{dt}\right)^2 + 9\left(\frac{dx}{dt}\right)^2}$

$2\sqrt{10} = \sqrt{10\left(\frac{dx}{dt}\right)^2}$

$2 = \sqrt{\left(\frac{dx}{dt}\right)^2}$ $\frac{dx}{dt} = 2$

$\frac{dy}{dt} = 3\frac{dx}{dt}$
$= 3(2)$
$= 6$

From 2003 BC Multiple Choice

4. For $0 \leq t \leq 13$, an object travels along an elliptical path given by the parametric equations $x = 3\cos t$ and $y = 4\sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

(A) $-\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $-\frac{4\tan 13}{3}$ (D) $\boxed{-\frac{4}{3\tan 13}}$ (E) $-\frac{3}{4\tan 13}$

$m = \left.\frac{dy}{dx}\right|_{t=13} = \left.\frac{dy/dt}{dx/dt}\right|_{t=13} = \left.\frac{4\cos t}{-3\sin t}\right|_{t=13} = \frac{4\cos 13}{-3\sin 13} = -\frac{4}{3}\cot 13 = -\frac{4}{3}\frac{1}{\tan 13}$

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84. A particle moves in the xy-plane so that its position at any time t is given by $x(t) = t^2$ and $y(t) = \sin(4t)$. What is the speed of the particle when $t = 3$?

(A) 2.909 (B) 3.062 (C) $\boxed{6.884}$ (D) 9.016 (E) 47.393

speed $= \sqrt{x'(t)^2 + y'(t)^2} = \sqrt{6^2 + 16(\cos 12)^2} = 6.884$

$x'(t) = 2t$ $x'(3) = 6$

$y'(t) = 4\cos 4t$ $y'(3) = 4\cos 12$

From 1998 BC Multiple Choice

21. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1 \sqrt{t^2+1}\,dt$ (B) $\int_0^1 \sqrt{t^2+t}\,dt$ $\boxed{\text{(C) } \int_0^1 \sqrt{t^4+t^2}\,dt}$ (D) $\frac{1}{2}\int_0^1 \sqrt{4+t^4}\,dt$ (E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2+9}\,dt$

$x' = t^2$ $\quad y' = t$

$(x')^2 = t^4$ $\quad (y')^2 = t^2$

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77. If f is a vector-valued function defined by $f(t) = (e^{-t}, \cos t)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$ (B) $-e^{-t} - \cos t$ (C) $(-e^{-t}, -\sin t)$

(E) $(e^{-t}, \cos t)$. $\boxed{\text{(E) } (e^{-t}, -\cos t)}$

$f'(t) = (-e^{-t}, -\sin t)$

$f''(t) = (e^{-t}, -\cos t)$

Answers

2008 Mult. Choice 1. B 5. C 28. D

2003 Mult. Choice 4. D 84. C

1998 Mult. Choice 21. C 77. E