KEY

MRS. HORVATH

AP Calculus
4.2 Worksheet

*All work must be shown in this course for full credit. Unsupported answers may receive NO credit.*

1. State the MVT 2 ways …

   a) … in words Somewhere between points A and B on a differentiable curve, there is at least one tangent line parallel to chord AB.

   b) … algebraically f(x) is continuous on [a,b] and differentiable on (a,b) then there is at least one point c in (a,b) at which $f'(c) = \frac{f(b)-f(a)}{b-a}$

2. Let $f(x) = -2x^2 + \underset{6}{14}x - 12$ on the interval [1, 6]

   a) How do you know this function satisfies the hypothesis of the MVT?

   Because this parabola is continuous on [1,6] and differentiable on (1,6)

   b) Find the value of $c$ guaranteed by the MVT.

$$-4c+14 = \frac{(-2(36)+84-12)-(-2+14-12)}{6-1} = \frac{0}{5}$$

$$-4c+14 = 0 \qquad c = \frac{14}{4} = \frac{7}{2} = \boxed{3.5}$$

$$-4c = -14$$

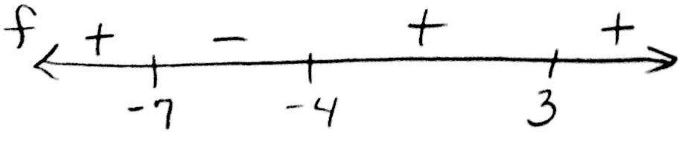
The last example is a special version of the Mean Value Theorem called Rolle's Theorem. In fact, the proof of the Mean Value Theorem can be done quite easily, if you prove Rolle's Theorem first. Rolle's Theorem basically states that if the function is continuous on the closed interval and differentiable on the open interval AND the values of the function at the endpoints are equal, then there must exist at least one point in the interval where the derivative is zero.

3. Summarize how we will use calculus to determine whether a function is increasing or decreasing.

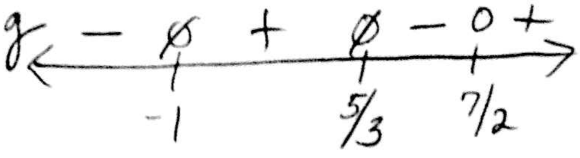
   If we show that a function's derivative is positive on (a,b) then the function is increasing on (a,b). Likewise, if derivative is negative, function is decreasing.

4. Make a sign chart for the following functions:

   a) $f(x) = (x-3)^2(x+4)(x+7)$



   b) $g(x) = \frac{5(2x-7)}{(x+1)(3x-5)}$



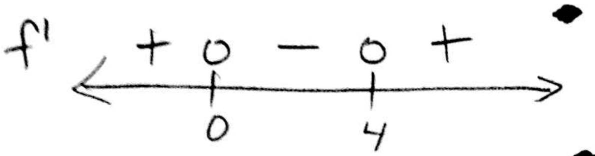
5. Find the critical numbers of $f$ and the intervals where $f$ is increasing or decreasing if $f(x)=x^3-6x^2+15$.

$f'(x)$ DNE? $f'$ exists for all real #s

$f'(x)=3x^2-12x$

$=3x(x-4)$

$f'(x)=0$? @ $x=0,4$



- f increasing on $(-\infty,0)$ and $(4,\infty)$ b/c $f'>0$ on those interva
- f decreasing on $(0,4)$ b/c $f'<0$ on that interval.

6. The Profit $P$ in dollars made by a fast food restaurant selling $x$ hamburgers is given by

$$P=2.44x-\frac{x^2}{20000}-5000,\quad 0\le x\le 35000.$$

a) Find the open intervals on which $P$ is increasing or decreasing

$P'=2.44-\frac{2}{20000}x=2.44-\frac{x}{10000}$

incr: $(0,24400)$

decr: $(24400,35000)$

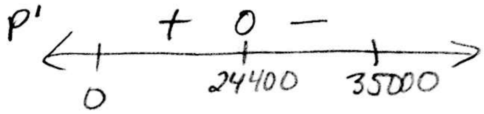
$P'$ exists for all reals

$P'=0$ when $2.44=\frac{x}{10000}$

$x=24400$

b) Find the maximum profit.

- Occurs @ $x=24400$ b/c $P'=0$ and
as from + to − there.



$P(24400)=2.44(24400)-\frac{(24400)^2}{20000}-5000$

$=\boxed{\$24{,}768}$

7. If you know that the acceleration of gravity is $-32\ ^{ft}/_{s^2}$, for an falling object, we could write the acceleration of the object at time $t$ as $a(t)=-32$.

a) Find a function for the velocity of the object at time $t$. What does the constant equal (in words)?

$v(t)=-32t+C$

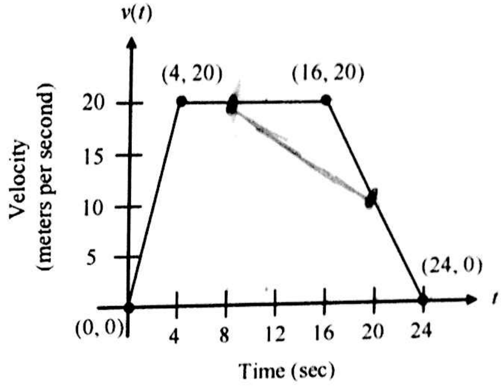
C equals the initial velocity (at $t=0$)

b) Find a function for the position of the object at time $t$. What does the constant equal (in words)?

$s(t)=-16t^2+C$

C equals the initial position (at $t=0$)

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by e piecewise-linear function defined by the graph below.



a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain its meaning.

*Obviously we haven't used this symbol yet, nor have we talked about how to et it ... so here's a couple of hints ...]*

i) *If I told you the notation* $\int_0^{24} v(t)\,dt$ *only asked you to find the antiderivative of the velocity function, you should be able to* *<u>use correct units</u>.*

s(t) in meters

ii) *If I told you that all the notation* $\int_0^{24} v(t)\,dt$ *means for this problem is to find the area under the given curve, you should then be able to answer the question AND* *<u>explain the meaning</u> of* $\int_0^{24} v(t)\,dt$.

Distance travelled from 0 to 24 seconds = area under curve

$= \frac{1}{2}(12+24)(20) = \boxed{360m}$

b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

$v'(4)$ = DNE b/c there is a corner there (LH deriv ≠ RH deriv)

$v'(20) = -\frac{20}{8} = \boxed{-\frac{5}{2}\ m/s^2}$

c) Let $a(t)$ be the car's acceleration at time $t$, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

$$a(t) = \begin{cases} 5, & 0<t<4 \\ 0, & 4<t<16 \\ -5/2, & 16<t<24 \end{cases}$$

d) Find the average rate of change of $v$ over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of $c$, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

avg rate of chg $= \frac{10-20}{20-8} = -\frac{10}{12} = -\frac{5}{6}$

NO, MVT does not apply because v(t) is <u>not</u> differentiable on entire interval $8<t<20$

[redacted]