FUNDAMENTAL THEOREM OF CALCULUS

Given $\frac{dy}{dx} = 3x^2 + 4x - 5$ with the initial condition $y(2) = -1$. Find $y(3)$.

$(2,-1)$

Method 1: Integrate $y = \int (3x^2 + 4x - 5)\,dx$, and use the initial condition to find C. Then write the particular solution, and use your particular solution to find $y(3)$.

$y = x^3 + 2x^2 - 5x + C$

$-1 = 8 + 8 - 10 + C$

$-1 = 6 + C$

$-7 = C$

$\boxed{y = x^3 + 2x^2 - 5x - 7}$

$y(3) = 27 + 18 - 15 - 7$

$\boxed{y(3) = 23}$

Method 2: Use the Fundamental Theorem of Calculus: $\int_a^b f'(x)\,dx = f(b) - f(a)$

$$y(3) = y(2) + \int_2^3 3x^2 + 4x - 5\,dx$$

$$= -1 + (x^3 + 2x^2 - 5x)\Big|_2^3$$

$$= -1 + (27 + 18 - 15) - (8 + 8 - 10)$$

$$= -1 + 30 - 6$$

$$\boxed{y(3) = 23}$$

Sometimes there is no antiderivative so we must use Method 2 and our graphing calculator.

Ex. $f'(x) = \sin(x^2)$ and $f(2) = -5$. Find $f(1)$.

$$f(1) = f(2) - \int_1^2 \sin(x^2)\,dx$$

$$= -5 - .4945\ldots$$

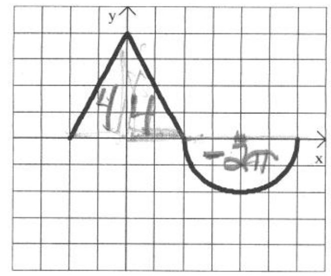
$$\boxed{f(1) = -5.495}$$

Ex. The graph of f' consists of two line segments and a semicircle as shown on the right. Given that $f(-2)=5$, find:

(a) $f(0) = f(-2) + \int_{-2}^{0} f'\,dx = 5 + 4 = \boxed{9}$

(b) $f(2) = f(0) + 4 = 9 + 4 = \boxed{13}$

(c) $f(6) = \boxed{13 - 2\pi}$



$\frac{\pi r^2}{2}$

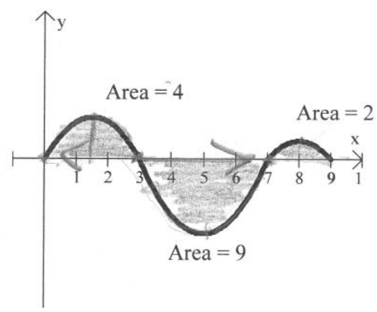
Graph of f'

Ex. The graph of f' is shown. Use the figure and the fact that $f(3)=5$ to find:

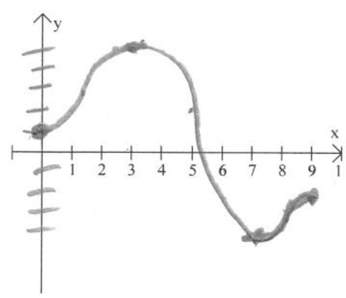
(a) $f(0) = 5 - 4 = \boxed{1}$

(b) $f(7) = 5 - 9 = \boxed{-4}$

(c) $f(9) = -4 + 2 = \boxed{-2}$



Then sketch the graph of f.



Ex. A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}$ °C per minute. Estimate the pizza's temperature when $t = 5$ minutes.

$R(5) = 95^\circ - \int_0^5 6e^{-.1t}\,dt = 95^\circ - 23.608^\circ$

$= \boxed{71.392^\circ C}$