FUNDAMENTAL THEOREM OF CALCULUS

Given $\frac{dy}{dx} = 3x^2 + 4x - 5$ with the initial condition $y(2) = -1$. Find $y(3)$.

<u>Method 1</u>: Integrate $y = \int (3x^2 + 4x - 5)\, dx$, and use the initial condition to find C. Then write the particular solution, and use your particular solution to find $y(3)$.

<u>Method 2</u>: Use the Fundamental Theorem of Calculus: $\int_a^b f'(x)\, dx = f(b) - f(a)$

Sometimes there is no antiderivative so we must use Method 2 and our graphing calculator.

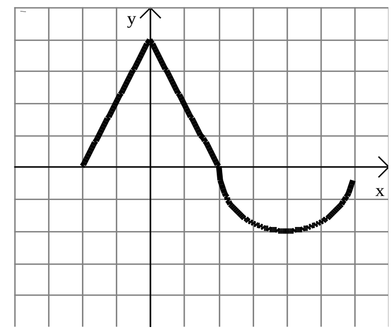
<u>Ex</u>. $f'(x) = \sin(x^2)$ and $f(2) = -5$. Find $f(1)$.

Ex. The graph of f' consists of two line segments and a semicircle as shown on the right. Given that $f(-2) = 5$, find:

(a) $f(0)$

(b) $f(2)$

(c) $f(6)$



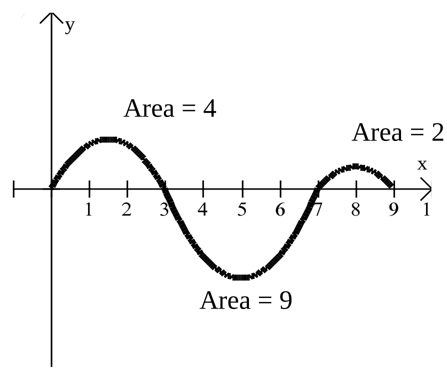
Graph of f'

Ex. The graph of f' is shown. Use the figure and the fact that $f(3) = 5$ to find:

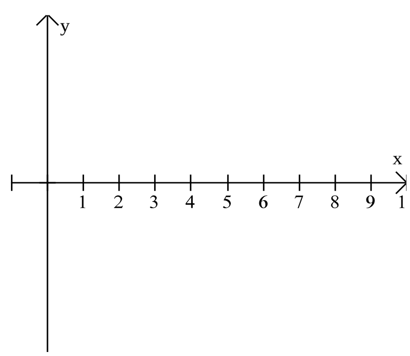
(a) $f(0)$

(b) $f(7)$

(c) $f(9)$



Then sketch the graph of *f*.



Ex. A pizza with a temperature of 95°C is put into a 25°C room when $t = 0$. The pizza's temperature is decreasing at a rate of $r(t) = 6e^{-0.1t}$ °C per minute. Estimate the pizza's temperature when $t = 5$ minutes.

CALCULUS
WORKSHEET 1 ON FUNDAMENTAL THEOREM OF CALCULUS

Work problems 1 - 2 by both methods. Do not use your calculator.

1. $y' = 2 + \frac{1}{x^2}$ and $y(1) = 6$. Find $y(3)$.

2. $f'(x) = \cos(2x)$ and $f(0) = 3$. Find $f\left(\frac{\pi}{4}\right)$.

Work problems 3 – 7 using the Fundamental Theorem of Calculus and your calculator.

3. $f'(x) = \cos(x^3)$ and $f(0) = 2$. Find $f(1)$.

4. $f'(x) = e^{-x^2}$ and $f(5) = 1$. Find $f(2)$.

5. A particle moving along the x-axis has position $x(t)$ at time t with the velocity of the particle $v(t) = 5\sin(t^2)$. At time $t = 6$, the particle's position is (4, 0). Find the position of the particle when $t = 7$.

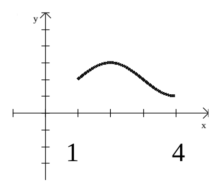
6. Let $F(t)$ represent a bacteria population which is 4 million at time $t = 0$. After t hours, the population is growing at an instantaneous rate of 2^t million bacteria per hour. Find the total increase in the bacteria population during the first three hours, and find the population at $t = 3$ hours.

7. A particle moves along a line so that at any time $t \geq 0$ its velocity is given by $v(t) = \dfrac{t}{1+t^2}$. At time $t = 0$, the position of the particle is $s(0) = 5$. Determine the position of the particle at $t = 3$.

Use the Fundamental Theorem of Calculus and the given graph.

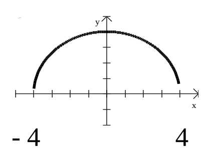
8. The graph of f' is shown on the right.

$\int_1^4 f'(x)\,dx = 6.2$ and $f(1) = 3$. Find $f(4)$.



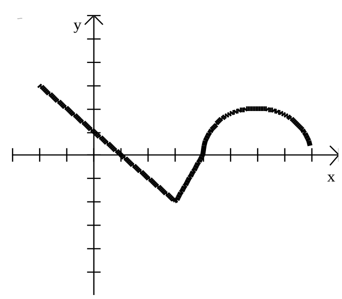
9. The graph of f' is the semicircle shown on the right.

Find $f(-4)$ given that $f(4) = 7$.

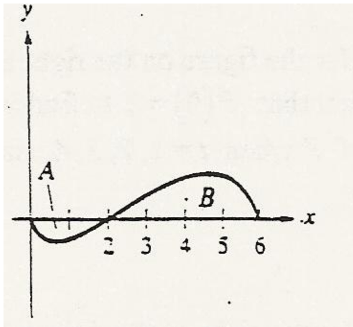


10. The graph of f', consisting of two line segments and a semicircle, is shown on the right. Given that $f(-2) = 5$, find:

(a) $f(1)$ (b) $f(4)$ (c) $f(8)$



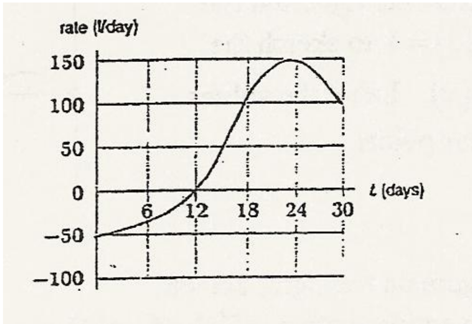
11. Region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Find:



(a) $\int_2^6 f(x)\,dx$

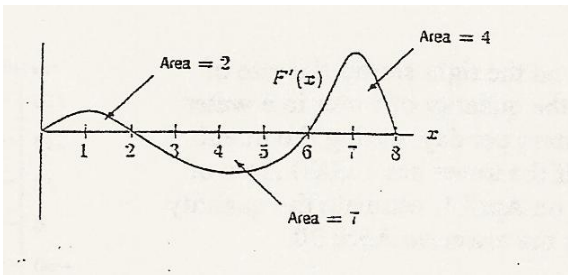
(b) $\int_0^6 |f(x)|\,dx$

12. The graph on the right shows the rate of change of the quantity of water in a water tower, in liters per day, during the month of April. If the tower has 12,000 liters of water in it on April 1, estimate the quantity of water in the tower on April 30.



13. A cup of coffee at 90° C is put into a 20° C room when $t = 0$. The coffee's temperature is changing at a rate of $r(t) = -7e^{-0.3t}\ °C$ per minute, with t in minutes. Estimate the coffee's temperature when $t = 10$.

14. Use the figure on the right and the fact that $F(2) = 3$ to sketch the graph of $F(x)$. Label the values of at least four points.



Answers

1. $\frac{32}{3}$

2. $\frac{7}{2}$

3. 2.932

4. 0.996

5. 3.837

6. 10.099 million, 14.099 million

7. 6.151

8. 9.2

9. $7-8\pi$

10. (a) 9.5 (b) 6.5 (c) $6.5+2\pi$

11. (a) 5 (b) 6.5

12. Answers will vary. With two triangles and a trapezoid: 13,350 liters; with two triangles and two trapezoids: 13,500 liters.

13. 67.828°C

CALCULUS
WORKSHEET 2 ON FUNDAMENTAL THEOREM OF CALCULUS

Use your calculator on problems 3, 8, and 13.

1. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)\,dx = 17$, what is the value of $f(4)$?

2. If $\int_2^5 (2f(x) + 3)\,dx = 17$, find $\int_2^5 f(x)\,dx$.

3. Water is pumped out of a holding tank at a rate of $5 - 5e^{-0.12t}$ liters/minute, where *t* is in minutes since the pump is started. If the holding tank contains 1000 liters of water when the pump is started, how much water does it hold one hour later?

4. Given the values of the derivative $f'(x)$ in the table and that $f(0) = 100$, estimate $f(x)$ for x = 2, 4, 6. Use a right Riemann sum.

x	0	2	4	6
$f'(x)$	10	18	23	25

5. Consider the function f that is continuous on the interval $[-5, 5]$ and for which $\int_0^5 f(x)\,dx = 4$. Evaluate:

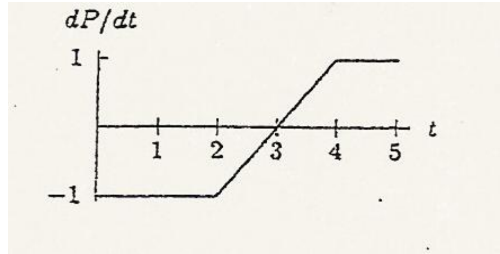
(a) $\int_0^5 (f(x) + 2)\,dx =$

(b) $\int_{-2}^3 f(x+2)\,dx =$

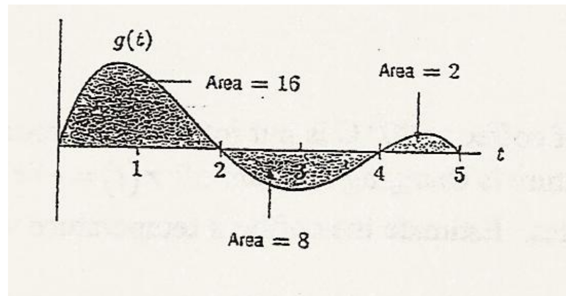
(c) $\int_{-5}^5 f(x)\,dx$ (f is even) $=$

(d) $\int_{-5}^5 f(x)\,dx$ (f is odd) $=$

6. Use the figure on the right and the fact that $P(0) = 2$ to find values of P when $t = 1, 2, 3, 4,$ and 5.



7. Using the figure on the right, sketch a graph of an antiderivatives $G(t)$ of $g(t)$ satisfying $G(0) = 5$. Label each critical point of $G(t)$ with its coordinates.



8. Find the value of $F(1)$, where $F'(x) = e^{-x^2}$ and $F(0) = 2$.

9. Given $f(x) = \begin{cases} 2x, & x \le 1 \\ 2, & x > 1 \end{cases}$. Evaluate: $\int_{1/2}^{5} f(x)\,dx$.

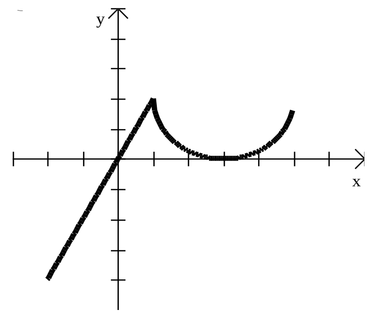
10. A bowl of soup is placed on the kitchen counter to cool. The temperature of the soup is given in the table below.

Time t (minutes)	0	5	8	12
Temperature $T(x)$ (°F)	105	99	97	93

(a) Find $\int_{0}^{12} T'(x)\,dx$.

(b) Find the average rate of change of $T(x)$ over the time interval $t = 5$ to $t = 8$ minutes.

11. The graph of f' which consists of a line segment and a semicircle, is shown on the right. Given that $f(1)=4$, find:



(a) $f(-2)$

(b) $f(5)$

12. (Multiple Choice) If f and g are continuous functions such that $g'(x)=f(x)$ for all x, then $\int_2^3 f(x)\,dx=$

(A) $g'(2)-g'(3)$ (B) $g'(3)-g'(2)$ (C) $g(3)-g(2)$

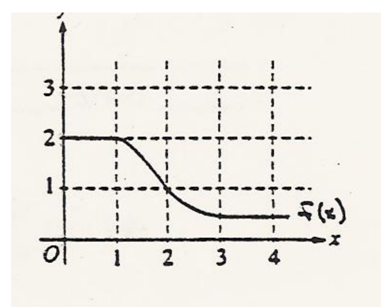
(D) $f(3)-f(2)$ (E) $f'(3)-f'(2)$

13. (Multiple Choice) If the function $f(x)$ is defined by $f(x)=\sqrt{x^3+2}$ and g is an antiderivatives of f such that $g(3)=5$, then $g(1)=$

(A) -3.268 (B) -1.585 (C) 1.732 (D) 6.585 (E) 11.585

14. (Multiple Choice) The graph of f is shown in the figure at right. If $\int_1^3 f(x)\,dx=2.3$ and $F'(x)=f(x)$, then $F(3)-F(0)=$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3



Answers

1. 29

2. 4

3. 741.636 liters

4. 136, 182, 232

5. (a) 14 (b) 4 (c) 8 (d) 0

6. 1, 0, $-\frac{1}{2}$, 0, 1

7. 21, 13, 15

8. 2.747

9. $8\frac{3}{4}$

10. (a) -12°F (b) $-\frac{2}{3}$°F/min.

11. (a) 7 (b) $12-2\pi$

12. C

13. B

14. D